then, expanding, and removing the term involving x^3 (by the proper linear transformation) the coefficient of x also vanishes, and from this property we find that the relation between the coefficients of (9) is :

$$p^3-4pq+8r=0,$$

and in (10), the sum of two of the roots is equal to the sum of the other two.

(Cf. Burnside and Panton's *Theory of Equations*, page 41).

2nd Case. Assume that (9) can be put in the form

$$(mx+\frac{n}{x})^2+g(mx+\frac{n}{x})+f=0\ldots.(11).$$

Multiplying out, and comparing with (9), we find that the relation between the coefficients is $r^2=p^2s$, *and the roots of* (11) *are in geometrical progression.*

Also solved by *CHARLES C. CROSS, B. L. REMICK, J. SCHEFFER, H. C. WHITAKER*, and *J. W. YOUNG.*

116. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Solve the equations :

$$w(xy+yz+xz)=a\ ;\ x(wy+wz+yz)=b\ ;$$

$$y(wx+wz+xz)=c\ ;\ z(wx+wy+xy)=d.$$

I. Solution by G. B. M. ZEZR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; J. M. HOWIE, The Nebraska State Normal School, Peru, Neb.; B. L. REMICK, Bradley Polytechnic Institute, Peoria, Ill.; NOAH ADAIR, A. M., Tyler College, Tyler, Tex.; G. I. HOPKINS, High School, Manchester, N.H.; FREMONT CRANE, B. S., C. E., Stockett, Mont.; H. C. WHITAKER, Ph.D., Manual Training School, Philadelphia, Pa., C. C. CROSS, Meredithville, Va.; L. B. FILLMAN, Chester, Pa.; O. S. WESTCOTT, A. M., Sc. D., North Division High School, Chicago, Ill.; and W. W. LANDIS, A. M., Dickinson College, Carlisle, Pa.

(1)+(2)+(3)+(4) gives $wxy+wxz+wyz+xyz=\frac{1}{3}(a+b+c+d)\ldots.(5)$.

(5)−(1) gives $xyz=\frac{1}{3}(b+c+d-2a)\ldots.(6)$.

(5)−(2) gives $wyz=\frac{1}{3}(a+c+d-2b)\ldots.(7)$.

(5)−(3) gives $wxz=\frac{1}{3}(a+b+d-2c)\ldots.(8)$.

(5)−(4) gives $wxy=\frac{1}{3}(a+b+c-2d)\ldots.(9)$.

(6)÷(7) gives $x/w=(b+c+d-2a)/(a+c+d-2b)$.

(6)÷(8) gives $y/w=(b+c+d-2a)/(a+b+d-2c)$.

(6)÷(9) gives $z/w=(b+c+d-2a)/(a+b+c-2d)$.

These values of x, y, and z in (1) give

$$w=\sqrt[3]{\frac{(a+b+c-2d)(a+b+d-2c)(a+c+d-2b)}{3(b+c+d-2a)^2}}=\sqrt[3]{\frac{BCD}{3A^2}},\text{ suppose.}$$

Similarly, $x=\sqrt[3]{(ACD/3B^2)}$, $y=\sqrt[3]{(ABD/3C^2)}$, $z=\sqrt[3]{(ABC/3D^2)}$.

II. Soluiion by J. SCHEFFER, A. M., Hagerstown, Md.; M. A. GRUBER, A. M., War Department, Washington, D. C.; J. H. DRUMMOND, LL. D., Portland, Me.; ELMER SCHUYLER, B. Sc., Boys' High School, Reading, Pa.; COOPER D. SCHMITT, M. A., University of Tennessee, Knoxville, Tenn.; and H. S. VANDIVER, Bala, Pa.

Putting $xyzw=P$, the given equations change into the following:

$$P/z+P/y+P/x=a,\ P/z+P/y+P/w=b,$$

$$P/z+P/x+P/w=c,\ P/y+P/x+P/w=d.$$

Adding, we get $P/x+P/y+P/z+P/w=\dfrac{a+b+c+d}{3}$.

Putting $\dfrac{a+b+c+d}{3}=s$, and subtracting each of the above equations, gives

$$P/w=s-a,\ P/x=s-b,\ P/y=s-c,\ P/z=s-d.$$

Multiplying, $P^3=(s-a)(s-b)(s-c)(s-d)$.

$$\therefore\ x=\sqrt[3]{\frac{(s-a)(s-c)(s-d)}{(s-b)^2}},\ y=\sqrt[3]{\frac{(s-a)(s-b)(s-d)}{(s-c)^2}},$$

$$z=\sqrt[3]{\frac{(s-a)(s-b)(s-c)}{(s-d)^2}},\ w=\sqrt[3]{\frac{(s-b)(s-c)(s-d)}{s-a)^2}}.$$

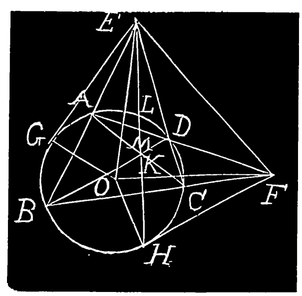
GEOMETRY.

146. Proposed by H. R. HIGLEY, M. Sc., Professor of Mathematics, Normal School, East Stroudsburg, Pa.

If the opposite sides of a quadrilateral inscribed in a circle be produced to meet, the square on the line joining the points of concurrence=the sum of the squares on the two tangents from these points. Ex. 24, page 219, Mackay's *Elements of Euclid*.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

Let $ABCD$ be the inscribed quadrilateral; E, F the intersection of the opposite sides produced; O the center of the circle; M the intersection of the diagonals; EG tangent from E; FH, FL tangents from F.



Draw EF, EM and let EM cut FO in K. It has been shown that EM is the polar of F.

$\therefore$ EM passes through L, H and is perpendicular to OF.

$\therefore$ $EF^2=FK^2+FK^2$.

But $EK^2=EO^2-OK^2$, $FK^2=(FO-OK)^2$.

$\therefore$ $FK^2=FO^2-2FO.OK+OK^2$.

But $2FO.OK=2OH^2=2r^2$. $\therefore$ $EK^2+FK^2=EO^2+FO^2-2r^2$.

$EG^2=EO^2-r^2$, $FH^2=FO^2-r^2$.

$\therefore$ $EG^2+FH^2=EO^2+FO^2-2r^2=EK^2+FK^2=EF^2$.